A GRAPHICAL METHOD TO DETERMINE OPERATING TIME OF RELAYS

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ABSTRACT

Saturation of the magnetic system results in non-linearity between the excitation current (which includes the operating current) and the induced voltage across the excitation winding. This hinders the use of analytical methods in calculating the value of operating time in relays because the accuracy involved is limited. This paper proposes and verifies a better and accurate graphical method to determine the operating time hence improving the accuracy involved when analytical methods cannot be used due to non-linearity of magnetizing curve.

INTRODUCTION

Due to difficulties in taking into account saturation in the magnetic system, magnetic leakages, the effect of eddy currents, change of load and the motion time of the armature; analytical methods for calculating operating times of relays have proved complicated and with limited accuracy[1]. Hence to obtain operating times of relays a more convenient method, which is of graphical nature and with acceptable accuracy, is proposed.

Operating time of a relay can be considered as a sum of start-off time and motion time. Start-off time is the interval of time for the increase of the relay magnetic flux from zero value to a critical value at which the armature starts moving. Motion time is the interval of time for the movement of the relay armature from the instant it starts to move to the instant it fully closes or fully opens. The paper considers obtaining both intervals graphically.

START - OFF TIME

Magnetic flux in the core increases comparatively slower and therefore the influence of eddy currents operating time can be neglected. In this case start-off time of the armature during operation of the relay can be calculated using the following equation:

$$\Phi = \Phi_{ss}\left(i - e^{-\frac{t}{\tau}}\right) \tag{1}$$

Where Φ - nstanteneous flux at a given instant t

i - instante4neous current at instant t

Φ_{ss} - steady state flux

τ - time constant

Assume that the armature starts moving at an instant when magnetic flux reaches a value $\Phi = \Phi_{op}$ while the time using equation (1) equals start-off time:

$$\Phi_{op} = \left(\Phi_{ss} - \Phi_{ss}e^{-\frac{t'_{so}}{\tau}}\right)$$

where

$$t'_{so} = \tau \ln \frac{\Phi_{ss}}{\Phi_{ss} - \Phi_{op}} \tag{2}$$

Dividing equation (2) by Φ_{op} a formula for calculating star-off time is obtained as:

$$t'_{so} = \tau \ln \frac{K_1}{K_1 - 1} \tag{3}$$

where

$$\ln \frac{K_1}{K_1 - 1} - cons\tan t\ \ effect; K_1 - safety\ \ factor$$

If there is a series connected resistor, start-off time using equation (3) becomes:

$$t'_{so} = \frac{L}{T} \ln \frac{K_1}{K_1 - 1} \tag{4}$$

Multiplying both the numerator and the denominator of equation (4) by I^2 and substituting L by K_0N^2 results into;

$$t_{so}^{'} = \frac{I^2 K_o N^2}{I^2 R} \ln \frac{K_1}{K_1 - 1} = \frac{K_o (IN)_{op} K_1^2}{P} \ln \frac{K_1}{K_1 - 1} \qquad (5)$$

where P = Power consumed; $(IN)_{op}$ = operating mmf of relay.

From the above formula it can be seen that for a given safety factor K_1 and a defined constant load I, start-off time of the relay is inversely proportional to the power consumed.

Effect of Saturation in the Steel Core

To estimate the start-off time taking into account the non-linearity of the magnetization curve it is proposed to use a graphical integration

$$U = iR + \frac{d\psi}{dt} \qquad (6)$$

Equation (6) results into:

$$dt = \frac{N}{U - iR} d\Phi$$

If a curve of Φ against IN is used, it is possible to construct Φ against 1/(IN-iN)[2]. The relationship is shown in figure 1. Time within which flux Φ changes from O to Φ_{op} is given by:

$$t_{so}^{'} = \int_0^{\Phi_{op}} \frac{N}{U - iR} d\Phi = \frac{N^2}{R} \int_0^{\Phi_{op}} \frac{d\Phi}{IN - iN}$$

The result of the above integral is equal to the area S is enclosed between the axes and the curve (figure 1). The size of this area multiplied by N^2/R taking into account scales Φ and 1/(iN) gives the start-off time of the relay.

MOTION TIME OF THE RELAY

The motion of the relay can be described as follows[3]:

$$m_1 \frac{dX^2}{dt^2} + r\frac{dX}{dt} + aX + F_o = F_{arm} \qquad (7)$$

where

m_1	-	mass of the moving parts of the relay
X	-	displacement of the relay
r	-	full motion resistance coefficient (proportional to speed) of armature and moving parts of the relay
a	-	relative rigidity of contact and opposing springs
F_o	-	initial value of opposing force
F_{arm}	-	resultant force acting on the armature.

By neglecting the initial opposing force ($F_o = 0$) and the frictional force against the motion ($rX' = 0$), equation (7) may be rewritten as follows:

$$F_{arm} dX = d\frac{m_1 V^2}{2} + F_m dX \qquad (8)$$

where:

$V = dX/dt$ - speed of the armature relative a point within the mass

$F_m = AX$ - force opposing the movement of armature

On the other hand, the transients in the relay winding may be expressed as follows:

$$U = iR + L\frac{di}{dt} + i\frac{dL}{dt} = iR + \frac{d\psi}{dt} \qquad (9)$$

Equations (8) and (9) are non-linear. The approximate solution of these two equations may be obtained by a graphical method involving a series of approximations.

Using equations (8) and (9) in final differences

$$U = iR + \frac{\Delta\psi}{\Delta t} \qquad (10)$$

$$F_{arm}\Delta X = \Delta\left(\frac{m_1 V^2}{2}\right) + F_m dX \qquad (11)$$

where $\Delta\psi$, Δt and $\Delta(m_1 V^2/2)$ are the final differences of the flux within the winding, the time of motion, and the kinetic energy of the armature corresponding to a small displacement of the armature ΔX.

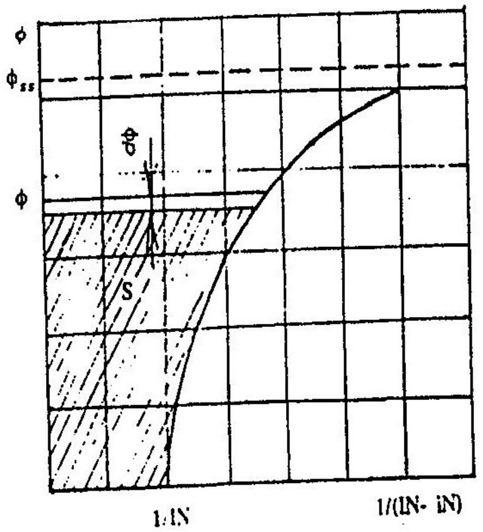

Fig. 1: The relationship between the flux ø and the value 1/(IN-iN)

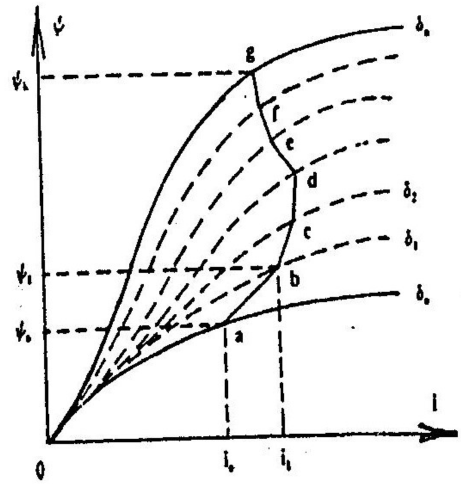

Fig. 2: The relationship between the flux within the core and the current through the relay winding when the armature is in motion

From point a in figure (2), corresponding to the start-off current, a line is drawn to the point of interception with a neighbourhood curve for airgap $\delta_1 = \delta_o - \Delta X$ such that ψ increases. Then obtain area OabO is equal to a work done $W_{arm} = F_{arm} \Delta X_1$ where F_{arm} - is average force acting within the interval from δ_o to δ_1. From equation (10) the armature speed V_1, at the end of the first interval can be obtained. The value F_m dX is obtained from the mechanical characteristics of the relay.

If we assume that the acceleration is a constant, the armature speed at the first interval is:

$$\Delta t_1 = \frac{\Delta X_1}{V_{av1}} \qquad (12)$$

$$V_{av1} = \frac{V_o + V_1}{2} . \frac{V_1}{2}$$

From figure 2 the elements of flux and current at the first interval can be obtained as follows:

$$\Delta\psi_1 = \psi_1 - \psi_o ; \Delta i_i = i_1 - i_o \quad (13)$$

Substituting in equation (10) the values $i_{av} = i_0 + \Delta i_i/2$, $\Delta \psi_1$ and Δt_1 the choice of interval ab can be proved right. If equation (10) is not satisfied new direction of line ab has to be chosen and the calculation repeated until the equation is found satisfied.

By a similar procedure the motion time of the armature at the second, third, and the rest of the intervals can be obtained. Total motion time is the sum of motion times for the individual intervals:

$$t_{mot} = \Delta t_1 + \Delta t_2 + \ldots + \Delta t_n \quad (14)$$

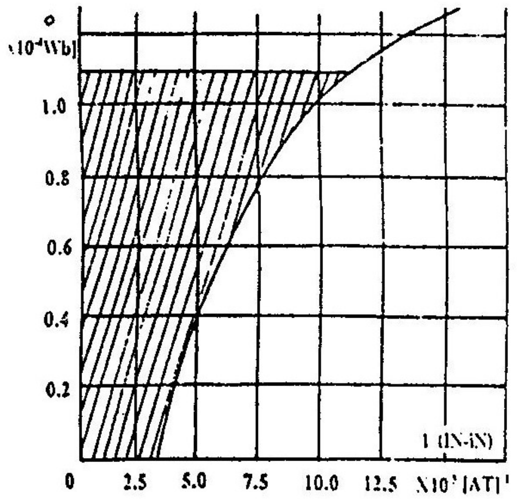

Fig. 3: The graph used to obtain the start-off time for the drop type relay.

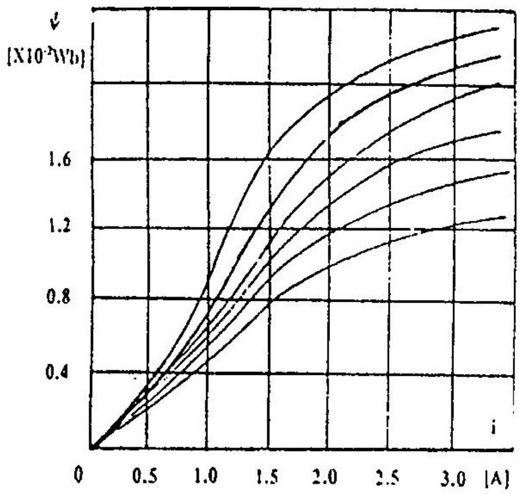

Fig. 4:The graph used to obtain the motion time for the drop type relay

RESULTS

Two types of relays were chosen to test the accuracy of the method, one a drop-type relay and the other solenoid type. Figures 3 and 4 show the graphs corresponding to the drop-type relay which were used with the proposed graphical method. The equivalent graphs for the solenoid type are shown in figures 5 and 6. The results obtained for both relays are tabulated below:

Table 1: Results obtained by using a drop-type relay

	Start-off time	Motion time	Operating time
Analytical (mSec)	3.60	11.96	15.56
Graphical (mSec)	3.54	11.90	15.44
Experimental (mSec)	-	-	15.90
Deviation of graphical (%) (compared to exp.)	-	-	2.89

Table 2: Results obtained by using a solenoid-type relay

	Start-off time	Motion time	Operating time
Analytical (mSec)	2.31	5.01	7.32
Graphical (mSec)	2.22	5.07	7.29
Experimental (mSec)	-	-	7.45
Deviation of graphical (%)(compared to exper.)	-	-	2.14

It is noted that it was not possible to obtain start-off time and motion time separately experimentally.

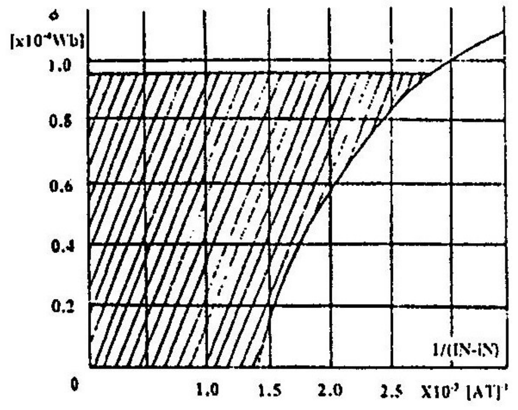

Fig 5: The graph for the solenoid type relay used to obtain start-off time

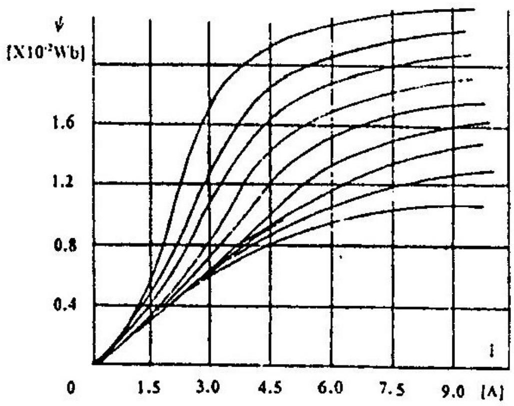

Fig. 6:The graph for the solenoid type used to obtain motion time

CONCLUSION

The graphical method takes into account the magnetic saturation whereas the analytical method ignores the saturation. With these limitations the proposed graphical method delivers acceptable accuracy and is advantegeous whenever saturation has to be taken into account. Through the results, accuracy has also been qualified.

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